Inelastic seismic torsional response of simple symmetric structures

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The inelastic seismic response of simple single-story structures, symmetric in plan in the elastic domain but having lateral load resisting structural elements (LLRSEs) of unequal yield strengths, has been studied. When yielding is initiated in such structures, an instantaneous state of torsional coupling in plan induces an additional torsional component to the response of the system. This torsional effect produces, under some circumstances, a magnification of the ductility demand of the LLRSE having lesser strength as compared to what would otherwise be expected of a single-degree-of-freedom. A parametric study has been conducted to investigate the circumstances where this amplification becomes significant, and the results of this research are presented herein. The rotational inertia is shown to have a significant influence on this transient state of inelastic torsional response. A limited investigation of single-story multi-element structures, single-story structures with complex force–displacement relationships, and simple multistory structures demonstrates that the findings and observations noted from the parametric study are equally applicable to more elaborate structures. Implications on Canadian design practice are discussed.

Key words: seismic response, torsional coupling, structural symmetry, inelastic response, ductility demand, parametric study, code implications, rotational inertia.

Une étude de la réponse sismique plastique de structures symétriques simples à un seul étage dans le domaine élastique, dotées d'éléments structuraux destinés à résister aux charges latérales et ayant des limites conventionnelles d'élasticité inégales, a été entreprise. Lorsqu'un accroissement de la déformation se produit dans ces structures, un état instantané de couplage de torsion en plan provoque un effet de torsion additionnel dans la réponse du système. Dans certaines circonstances, cet effet de torsion entraîne une amplification de la demande en ductilité de l'élément structural présentant une résistance plus faible aux forces latérales, en comparaison de ce que l'on peut s'attendre d'un élément à un seul degré de liberté. Une étude paramétrique a été entreprise afin d'étudier le contexte dans lequel cette amplification devient importante et cet article en présente les résultats. Il est démontré que l'inertie de rotation influe grandement sur le régime transitoire de la réponse en torsion plastique. Une étude limitée de structures à un seul étage et à éléments multiples, de structures à un seul étage présentant des relations force–déplacement complexes et de structures simples à plusieurs étages, démontre que les résultats et les observations provenant de l'étude paramétrique sont également applicables à des structures plus complexes. Les effets sur les pratiques canadiennes de conception sont également discutés.

Mots clés : réponse sismique, couplage de torsion, symétrie structurale, réponse inélastique, demande en ductilité, étude paramétrique, implications au niveau du code, inertie de rotation.

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Introduction

Torsional coupling in plan is introduced in a structure when the stiffness distribution of the lateral load resisting structural elements (LLRSEs), the mass, or both, results in a non-coincidence between the centers of mass and the centers of stiffness. Torsionally coupled structures are prone to severe seismically induced damage as demonstrated by many large earthquakes, including the Mexico City earthquake of 1985 (Mitchell *et al.* 1986). Often, during the structural seismic adequacy evaluation of an existing building, seismic survivability may be found to be seriously impaired by the presence of large eccentricities in plan. In such a case, the logical seismic retrofitting strategy would consist of adding new LLRSEs such as to eliminate these eccentricities. While the resulting retrofitted structure becomes symmetric in the linear elastic sense, the new structural elements used are often of a different type than the existing ones. This possibly results in dissimilar yield strengths and dissimilar force–displacement relationship between the various LLRSEs, and a "transient" state of torsional response will consequently develop as the structure is excited in the inelastic range. It is noteworthy that such dissimilarities can also be present in many types of new or existing structures, simply as a consequence of other engineering or architectural decisions; thus the particular structural behavior described above can be equally attributable to various other causes.

Consequently, the authors have conducted a research program to investigate the effect of numerous parameters on the transient inelastic torsional response of such structures and to identify which conditions lead to unsatisfactory inelastic performance. In a first approach to this broad subject, the present study was limited to the consideration of idealized LLRSEs of identical force–displacement relationship having dissimilar yield strengths. In the aforementioned retrofitting perspective, this would imply that the new LLRSEs are not overwhelmingly more rigid than the existing ones, but are rather introduced to correct structural deficiencies partly due to excessive plan eccentricities, i.e., the existing elements would still contribute to the total seismic resistance and are assumed capable of sustaining numerous cycles of large inelastic deformations.

This paper presents the results of a parametric study of over 2400 different simple initially symmetric structures having two LLRSEs. Ductility demand of the edge LLRSEs was selected to be the response parameter of interest in this study. It was compared with the ductility demand obtained for an equivalent single-degree-of-freedom (SDOF) system under identical seismic excitation. The results, presented in a normalized form, were obtained from time-history nonlinear analyses conducted for five different earthquake records; statistical means and standard deviations of the key response parameters were calculated. The findings from this limited study provide some preliminary criteria to assess the significance of dissimilar yield strengths on the inelastic seismic torsional response of initially symmetric systems. Furthermore, the results from additional case studies investigating the behavior of more complex structures are also reported herein, and compared with those from the simpler structures.

It must be emphasized that at the onset of the currently reported research program, the authors were unaware of research concentrating on the inelastic plan response of structures having considerable dissimilarities in yield strengths or force–displacement relationships, although other researchers had investigated the effect of torsional instability of symmetric systems with LLRSEs sharing identical strength and force–displacement relationship (Antonelli *et al.* 1981; Pekau and Syamal 1984; Tso 1975; Tso and Asmis 1971), and the effect of seismic wave motions characteristics in exciting torsional modes in otherwise symmetric structures (Awad and Humar 1984; Newmark 1969). The research described herein was performed as part of the first author's Ph.D. thesis in 1987 (Bruneau 1987). In the time elapsed since, similar work has been published by Pekau and Guimond (1990). Both pieces of work are complementary, as they address the same problem in a different perspective. As the problem of inelastic seismic response of torsionally coupled structures is currently receiving a renewed and considerable attention by the research community, it is felt that the present reporting of the original work by the authors is worthwhile, and long overdue.

Equations of motion around the center of mass

The general linear elastic equations of motion around the center of mass for single-story torsionally coupled systems are well known and have been derived by others (Awad and Humar 1984, among many). Although these apply to structures initially having eccentricities in plan, their presentation is useful to define essential parameters referred to herein, and set the problem in its proper perspective.

For monosymmetric systems (that is, systems having at least one axis of symmetry) and neglecting torsional seismic excitation, the equations along the y-axis (axis of symmetry) are decoupled, and the resulting coupled translational-torsional equations of motion are simplified to the following:

$$[1] \quad \begin{bmatrix} m & 0 \\ 0 & mr^2 \end{bmatrix} \begin{bmatrix} \ddot{v}_x \\ \ddot{v}_\theta \end{bmatrix} + \begin{bmatrix} K_x & -K_x e \\ -K_x e & K_\theta \end{bmatrix} \begin{bmatrix} v_x \\ v_\theta \end{bmatrix} = -\begin{bmatrix} m\ddot{v}_{gx} \\ 0 \end{bmatrix}$$

and, equivalently,

$$[2] \quad \begin{bmatrix} \ddot{v}_x \\ r\ddot{v}_\theta \end{bmatrix} + \omega_x^2 \begin{bmatrix} 1 & -e/r \\ -e/r & \Omega^2 \end{bmatrix} \begin{bmatrix} v_x \\ rv_\theta \end{bmatrix} = \begin{bmatrix} -\ddot{v}_{gx} \\ 0 \end{bmatrix}$$

with

$$[3] \quad \Omega = \omega_\theta/\omega_x = T_x/T_\theta$$

$$[4] \quad \omega_x^2 = K_x/m$$

$$[5] \quad \omega_\theta^2 = K_\theta/mr^2$$

where K_x and K_θ are the system's translational (along x) and rotational (around θ) stiffnesses for the resulting two-degrees-of-freedom system, and e is the static eccentricity of this system, expressed by

$$[6] \quad K_x = \sum_i K_{ix}$$

$$[7] \quad K_\theta = \sum_i K_{ix} y_i^2 + \sum_j K_{jy} x_j^2$$

$$[8] \quad e = \frac{1}{K_x} \sum_i y_i K_{ix}$$

The mass of the floor is m and its radius of gyration is r; v_x and $\ddot{v}_x$ are the translational displacement and acceleration of the center of mass in direction x; v_θ and $\ddot{v}_\theta$ are the rotational displacement and acceleration of the floor around a vertical axis; $\ddot{v}_{gx}$ is the ground acceleration in direction x; y_i and x_j are the distances from elements i and j to the center of mass; and K_{ix} and K_{jy} are the translational stiffnesses of elements i and j in the x and y directions respectively. The translational and torsional uncoupled frequencies, ω_x and ω_θ, the corresponding uncoupled periods, T_x and T_θ, and the ratio of those uncoupled frequencies, Ω, are defined in equations [3]–[5]. The torsional stiffness of individual lateral load resisting elements is neglected. The inelastic seismic response of such systems having eccentricities in plan ($e \neq 0$) has been discussed elsewhere (Bruneau and Mahin 1990).

For linear elastic symmetric systems ($e = 0$), the equations become uncoupled, and the torsional response would typically not be calculated due to the absence of torsional seismic excitation in the above formulation. This uncoupling persists until dissymetric yielding of the LLRSEs occurs, at which point a transient state of torsional coupling is established. New instantaneous properties, K_x', K_θ', e', ω_x', ω_θ', and Ω', can be calculated at each instant, and substituted in equations [1]–[8] to obtain the corresponding instantaneous equations of motion. Although this is the procedure indirectly followed by nonlinear time-history structural analysis programs, the number of possible values that can be taken by each of these variables grows rapidly with the complexity of the structure and (or) force–displacement relationship used.

More recently, strength eccentricity, e_P, or plastic eccentricity, defined as the distance from the center of strength at ultimate to the center of mass, has been introduced as a new structural parameter to relate to the inelastic seismic response of torsionally coupled structures (Sadek and Tso 1988). Although at a given time during the earthquake excitation, e' may greatly exceed e_P, and for complex force–displacement relationships these values may never be equal, the plastic eccentricity can be a useful descriptive parameter. However, this concept is not used in the current study. Initial structural properties, along with explicit expres-

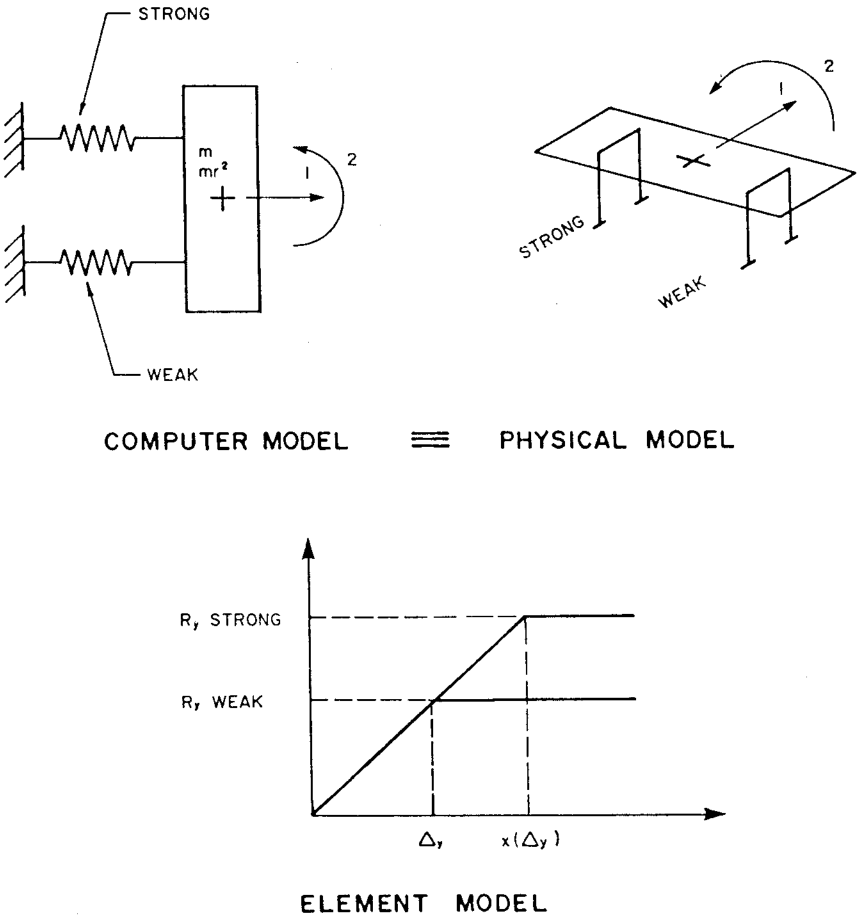

FIG. 1. Model of initially symmetric structure with two LLRSEs and bilinear force–displacement relationship used in this study.

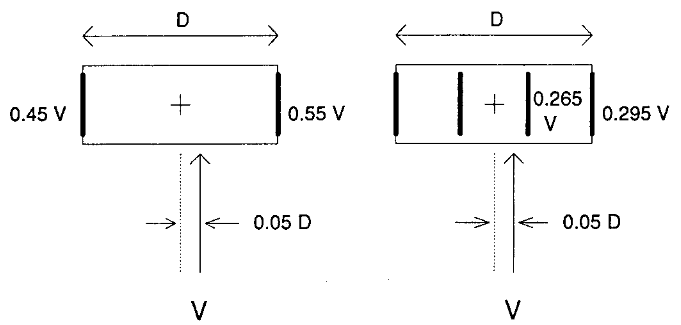

FIG. 2. Effect of redundancy and code-specified accidental eccentricity on element design.

sions of relative strengths, as defined in a following section, have been selected as the descriptive parameters.

In this study, simple structures having two LLRSEs are used to conduct a comprehensive parametric study. All floor diaphragms are assumed to be infinitely rigid in their own plane. Elements in the orthogonal direction are ignored for the sake of simplicity (i.e., $\Sigma K_{jy} x_j^2 = 0$). The system studied is illustrated in Fig. 1. Lateral load resisting elements are assumed herein to be equidistant from the center of mass.

Although secondary to this study and unrelated to the above model selection, it is noteworthy that many current building codes indirectly promote the reduction of plan redundancy. In an apparently symmetric building, the accidental eccentricity provision mandated by the equivalent static seismic lateral force design method of most building codes provides a minimum design eccentricity which is thought to account for uncertainties in mechanical properties, mass distribution, and ground motion. This accidental eccentricity is usually set by different codes to a small percentage (typically 5% or 10%) of the plan dimension perpendicular to the excitation. Abiding by those codes, should only two LLRSEs be present in each principal directions (Fig. 2*a*), an accidental eccentricity of 5% will increase the design forces in each element by 10%. If, instead, four equally spaced elements with equal stiffnesses are now considered (Fig. 2*b*), the same accidental eccentricity requirements will increase the design forces by 18% for the edge elements and by 6% for the inside elements, for a net increase of 12%, and, therefore, the more redundant structure is only achieved at a premium in material and labor. Consequently, strict adherence to building codes' seismic provisions would make the two-element structure a more economical design alternative, which is apparently discordant with earthquake engineering's traditional wisdom that redundancy improves the ultimate seismic resistance of structures.

Nonlinear analysis of initially symmetric system — parametric study

Methodology

A parametric study was performed to assess the significance of the torsional coupling developing in the inelastic

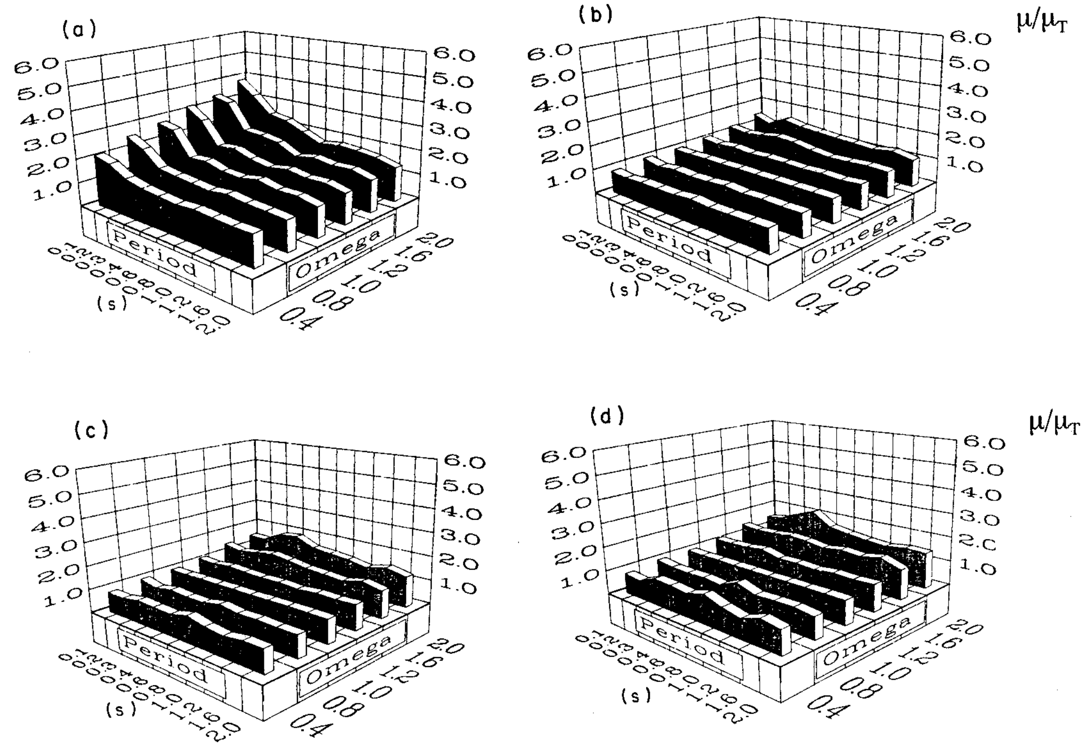

FIG. 3. Mean of weak LLRSE ductility amplification ratios for target ductility of 4; (*a*) 0.8 and 1.0 R_y, (*b*) 1.0 and 1.2 R_y, (*c*) 1.0 and 1.5 R_y, and (*d*) 1.0 and 2.0 R_y.

range when symmetric structures consist of LLRSEs of identical force–displacement relationship, but dissimilar yield strengths; in particular, the effect of various parameters on the element ductility demand of these initially symmetric structures were investigated when equivalent SDOF systems achieved preselected target displacement ductility values, μ_T.

For this parametric study, a simple bilinear inelastic element model was chosen. Here, ductility demand, μ, is defined as the maximum displacement, in absolute value, divided by the yield displacement. The introduction of more sophisticated modeling was not warranted at this stage; however, provided both LLRSEs share the same force–displacement relationship type, the element model has been found to have little influence on the conclusions of this study, as demonstrated in a following section.

Strain-hardening was set at 0.5% of the initial modulus of elasticity, making the element model almost elasto-perfectly plastic. LLRSEs in any given initially symmetric structure were modeled to have the same elastic stiffness (Fig. 1). The damping was chosen to be of the Rayleigh type, arbitrarily set at 2% of the critical damping for each of the true elastic frequencies of the systems analyzed. For the initially symmetric structures used in this study, the LLRSEs yield strength combinations are expressed as "R_y and $x(R_y)$," x being a fraction or multiplier of the reference yield strength, R_y. For $x \neq 1.0$, the resulting mismatch between the yield strengths of the LLRSEs produces the inelastic torsional response of interest in this study. The strong and weak elements are obviously defined as those having the larger and smaller yield strengths respectively. For $x = 1.0$, the resulting systems constitute equivalent SDOF systems whose inelastic responses provide a basis for comparing LLRSE ductility demands.

The study was performed for ten values of uncoupled period T_x (0.1, 0.2, 0.3, 0.4, 0.6, 0.8, 1.0, 1.2, 1.6, and 2.0 s), six values of the ratio of uncoupled frequencies Ω (0.4, 0.8, 1.0, 1.2, 1.6, and 2.0), two SDOF target ductility levels μ_T (4 and 8), and four LLRSE yield strength combinations (0.8 and 1.0 R_y, 1.0 and 1.2 R_y, 1.0 and 1.5 R_y, and 1.0 and 2.0 R_y). The equivalent SDOF systems used for comparison all yield at R_y.

The four chosen yield strength combinations bracket many possible situations. Ultimately, further increasing the strong LLRSE's strength would lead to permanent elastic response of the strong element with no further changes in torsional response. Note that although the difference in yield strength is herein assumed to result from the difficulty, or impossibility, in achieving similar yield levels in different LLRSEs, this difference also implicitly considers the difficulty in accurately predicting the yield strength of some types of structural systems. Further, the intent is to assess the significance of overestimating or underestimating the yield strength of one element, and consequently, structures of different ultimate translational strengths are compared in this process.

The following methodology was adopted for the parametric study:

1. SDOF systems were selected to have a period equal to the uncoupled translational frequency of their corresponding initially symmetric structure. These SDOF systems were designed such that they shared the same hysteretic characteristics and the same yield displacement, Δ_y. From the eigensolution of equation [2] when $e/r = 0$, the following relationships between the uncoupled periods (T_x and T_θ) and the true periods (T_1 and T_2) are obtained:

$$[9] \quad T_1 = T_x/\Omega, \qquad T_2 = T_x; \qquad \text{when } \Omega < 1.0$$

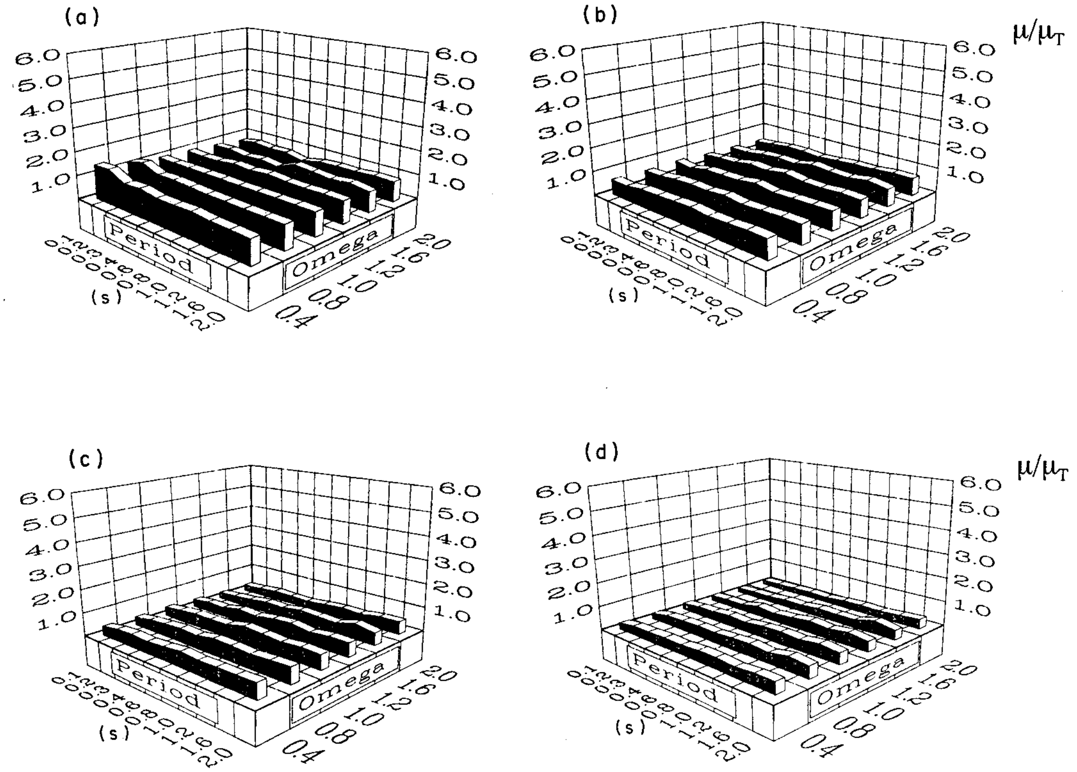

FIG. 4. Mean of strong LLRSE ductility amplification ratios for target ductility of 4; (*a*) 0.8 and 1.0 R_y, (*b*) 1.0 and 1.2 R_y, (*c*) 1.0 and 1.5 R_y, and (*d*) 1.0 and 2.0 R_y.

[10] $T_1 = T_x, \qquad T_2 = T_x/\Omega;$ when $\Omega > 1.0$

[11] $T_1 = T_2 = T_x;$ when $\Omega = 1.0$

However, in this study, the uncoupled translational period, T_x, is the only period of an initially symmetric structure excited before initiation of yielding, and the logical choice for the corresponding SDOF system's period.

2. Normalized strength factors necessary for each SDOF system to attain target ductilities, μ_T, of 4 and 8 were calculated for each earthquake record considered, using the program NONSPEC (Mahin and Lin 1983). Constant ductility inelastic response spectra were constructed, from which the required strength factors, as a function of SDOF period, were read. Normalized strength factors are defined as

[12] $$\eta = R_y/ma_{max} = K_x\Delta_y/ma_{max} = \omega_x^2\Delta_y/a_{max}$$

where a_{max} is the peak ground acceleration of a particular earthquake record, R_y is the yield strength of the SDOF system, and m is the mass of the equivalent SDOF system. For simplicity in this study, peak ground accelerations were scaled as necessary, for fixed values of element model properties, to satisfy the imposed target ductility condition. These steps were taken to ensure that the SDOF sytems were insensitive to variations in ground motion intensity. While this departs from a design approach, it ensures that any period-dependancy observed in the calculated ductility amplification ratios (see item 4 following) is only attributable to the inelastic torsional coupling phenomena, and not to the seismic input spectral characteristics.

3. For the earthquake excitation levels calculated in the previous step, the same systems were reanalyzed now considering the unequal LLRSE yield strengths. The maximum inelastic LLRSE displacements were then calculated, as well as the corresponding LLRSE ductility demands.

4. The ductility demands calculated for each initially symmetric case analyzed above were then divided by the ductility demands obtained from their respective equivalent inelastic SDOF system, to obtain a ratio of the ductilities (indicated as *ductility amplification ratios* in all figures herein). This amplification ratio provides a normalization over the selected target ductilities. It is believed that the ductility amplification ratios for each LLRSE of the two-element structures provide the best quantitative measure of the damage sensitivity of the systems.

Ductility demands of individual LLRSEs, as well as maximum displacement magnitudes if desired, can be both deduced from the ductility amplification ratios. For example, for an initially symmetric system having yield strengths R_y and $2R_y$, if ductility amplification ratios of 2.5 and 0.75 would have been obtained for the weak and strong LLRSEs respectively, ductility demands would then be $2.5\mu_T$ and $0.75\mu_T$, and maximum displacement magnitudes would be $2.5\mu_T\Delta_y$ and $0.75\mu_T2\Delta_y$ ($= 1.5\mu_T\Delta_y$) for the weak and strong LLRSEs respectively.

To provide results mostly independent of the particular characteristics of a single earthquake, five different earthquake records (El Centro 1940 N-S, Olympia 1949 N-S, Parkfield 1966 S16E, Paicoma Dam 1971 N65E, and Taft 1952 N21E) were considered, and the mean, and mean-plus-one-standard-deviation, of response values were calculated.

Response analyses for the initially symmetric structures were performed using the general nonlinear dynamic analysis program ANSR-1 (Mondkar and Powell 1975). The time step used in the time-history analyses using ANSR was chosen to

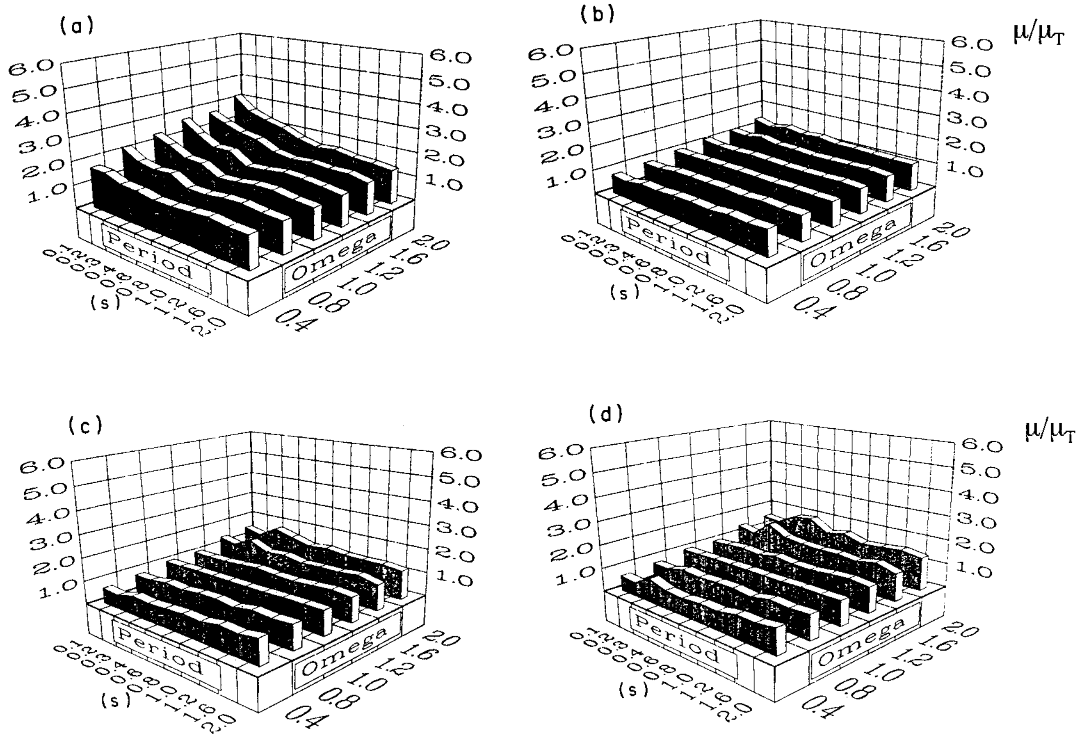

FIG. 5. Mean of weak LLRSE ductility amplification ratios for target ductility of 8; (*a*) 0.8 and 1.0 R_y, (*b*) 1.0 and 1.2 R_y, (*c*) 1.0 and 1.5 R_y, and (*d*) 1.0 and 2.0 R_y.

be at least less than $T/30$ of the smallest of the two true periods of each system, but never smaller than 0.002 s or larger than 0.02 s.

Results of the parametric study and observations

Figures 3–6 present the results from step 4 above. These plots show the mean ductility amplification ratios of the weak (Fig. 3) and strong (Fig. 4) LLRSEs for a target ductility of 4, followed by the same information for weak elements for a target ductility of 8 (Fig. 5), and the mean-plus-one-standard-deviation results for a target ductility of 4 (Fig. 6), for the five earthquake records selected. LLRSE's strength combinations of 0.8 and 1.0 R_y, 1.0 and 1.2 R_y, 1.0 and 1.5 R_y, and 1.0 and 2.0 R_y are identified as cases (*a*) to (*d*) respectively in each of these figures.

Considering the nature of ductility measurements in earthquake engineering, and the accuracy expected in ductility prediction of this kind, it might be said that LLRSE ductility amplification ratios of 1.25 or less are not considered significant, ductility amplification ratios from 1.25 to 1.5 are considered of moderate importance, and ratios above 1.5 are judged to be of major importance. Following this arbitrary convention, the following can be observed:

The weak LLRSE ductility amplification ratios for the case of 0.8 and 1.0 R_y are always at least of moderate importance, and often of major importance. This amplification is most severe for cases with small periods or large Ω values (and most significantly a combination of both), with ductility amplification ratios ranging up to 4 for the mean response (Figs. 3 and 5), and up to 5 for the mean-plus-one-standard-deviation response (Fig. 6). Amplifications were somewhat expected, since the ultimate translational strength of the $0.8R_y$ and R_y systems is less than that of their reference systems; nevertheless, the rather large magnification of weak LLRSE ductilities obtained remains impressive. For individual earthquake records, this amplification was sixfold at times, translating into a weak LLRSE ductility of almost 24 for the target ductility of 4. This is hardly a reasonable design ductility. Ductility amplification ratios of mostly moderate importance would be obtained instead, should the comparison be performed with a SDOF system with a $0.8R_y$ strength, as illustrated in Table 1, but large absolute ductilities would be obtained in both cases. While the overestimation of yield capacity was already known to be of significant consequences, it is seen to be even more so for the weak LLRSE of initially symmetric structures.

2. When the yield strength of one LLRSE is superior to that of the reference SDOF system, the weak LLRSE ductility amplification ratios are mostly non-affected until Ω becomes larger than 1.6 for the mean response, or 1.2 for the mean-plus-one-standard-deviation. In that case, the response is also seen to increase slightly along with the yield strengths differentials. The increase in weak LLRSE ductility amplification ratios, despite the increased ultimate translational strength of the systems, is surprising. It implies that the added torsional behavior induced by the increase in yield strength differential more than overcomes the benefit one might associate with the increase in strength (or balances it in the best case). Increases of 100% are seen for large Ω and large yield strength differences, and much larger ductility amplification ratios, often up to 2.5, were observed for single earthquake excitation results. Thus, there is no guarantee that an unbalanced increased strength in a symmetric structure decreases ductility demand. It should be noted that at some point, further increase in yield strength differential would produce no additional change in response for either LLRSE, as the strong element would reach permanently elastic behavior.

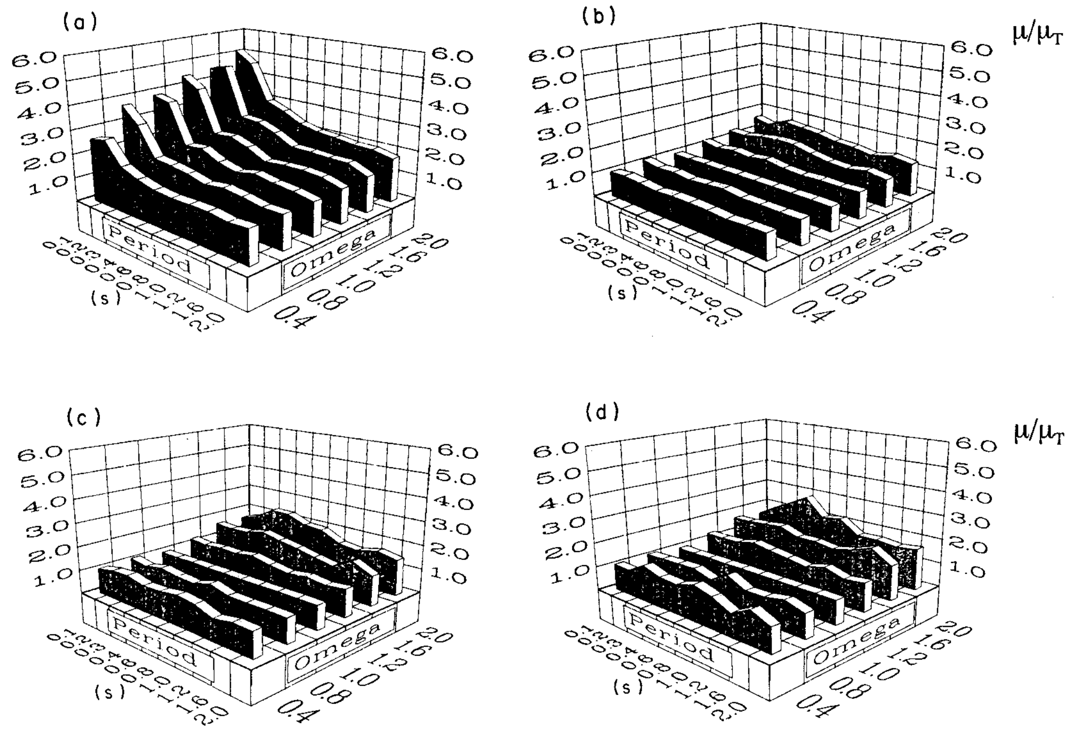

FIG. 6. Mean-plus-one-standard-deviation of weak LLRSE ductility amplification ratios for target ductility of 4; (*a*) 0.8 and 1.0 R_y, (*b*) 1.0 and 1.2 R_y, (*c*) 1.0 and 1.5 R_y, and (*d*) 1.0 and 2.0 R_y.

3. The strong LLRSE ductility amplification ratios are all less than 1.0, except in the 0.8 and 1.0 R_y cases where the lower ultimate translational strengths make increases in ductility demand also possible in the stronger LLRSE. Ductility amplification ratios of moderate importance can be noticed in the case of low periods ($T \leq 0.2$) and low Ω values ($\Omega \leq 0.8$) (Fig. 4). For the case of mean-plus-one-standard-deviation, these ratios are of moderate importance when $T \leq 0.6$ and $\Omega \leq 0.8$, and of major importance when $T \leq 0.2$ and $\Omega \leq 0.4$. Mean-plus-one-standard-deviation results for strong LLRSEs are presented elsewhere (Bruneau and Mahin 1987). For the other yield strength combinations, the strong LLRSE ductility amplification ratios is seen to reduce in proportion to the increase in yield strength differential. It is noteworthy that the decrease in strong LLRSE ductility amplification ratios occurring with the increase in the ultimate translational strength of the structures is partly a consequence of the increase of the yield strength of the strong LLRSE; i.e., an increase in yield strength (corresponding to an equivalent increase in the yield displacement) will produce an effective reduction in the ductility demand for a given magnitude of displacement. A value of the strong LLRSE ductility amplification ratio below 1.0 partly reflects that situation; it also does not imply that the strong LLRSE remains elastic, but simply that ductility demand is less than that of its corresponding SDOF system.

4. The observed ductility amplification ratios are generally independent of target ductility levels, i.e., the trends in response observed above (Fig. 3) remain unchanged for the higher target ductility case (Fig. 5). Changes are mostly of the order of 25%.

5. The structural period is seen to have no significant influence on the ductility amplification ratios of initially symmetric structures, except for the 0.8 and 1.0 R_y case, where weak LLRSE ductility amplification ratios are generally larger for structures with small periods (Figs. 3, 5, and 6). This is expected, as equivalent SDOF systems have been calibrated to target ductilities with yield strength set at R_y. The case of 0.8 and 1.0 R_y having smaller ultimate translational strength than its equivalent SDOF system, the natural tendency of short period structures to have larger response than more flexible ones, typical of earthquakes for the west coast of the United States, resurfaces. Again, referring to Table 1, where are presented the ductility demands of SDOF systems yielding at $0.8R_y$ but subjected to the scaled earthquake excitations used in the target ductility calibration of SDOF systems yielding at R_y, it can

TABLE 1. Displacement ductility demand (μ) of structures subjected to scaled seismic excitations that would produce $\mu = 4.0$ on SDOF systems yielding at $1.0R_y$

Period (s)	Displacement ductility demand, μ		Amplification ($\mu_{\text{weak element}}/\mu_{\text{SDOF}}$)
	SDOF yielding at $0.8R_y$	Weak element 0.8 and 1.0 R_y and $\Omega = 2.0$	
0.1	11.1	15.3	1.38
0.2	10.1	12.9	1.29
0.3	6.6	10.0	1.52
0.4	6.8	9.5	1.39
0.6	5.8	8.1	1.40
0.8	5.7	6.8	1.18
1.0	6.4	7.3	1.13
1.2	5.5	7.0	1.27
1.6	5.6	6.6	1.18
2.0	4.8	5.7	1.19

TABLE 2. LLRSE ductility amplification ratios for initially symmetric multi-element single-story structures having T_x = 0.4 s, and LLRSE strength combinations of 1.0 and 1.5 R_y

System characteristics		Ω					
LLRSE	Plan sequence of LLRSE*	0.4	0.8	1.0	1.2	1.6	2.0
	(a) Reference two-LLRSE structure						
Weak	WS	0.84	0.90	1.00	1.25	1.80	1.94
Strong	WS	0.59	0.57	0.73	0.69	0.34	0.45
	(b) Four equally spaced LLRSE structures						
Weak	WSSS	0.81	0.76	0.91	1.17	1.37	1.38
	WWSS	0.85	0.88	1.06	1.34	1.64	1.64
	WWWS	0.87	1.02	1.11	1.36	1.63	1.68
Strong	WSSS	0.55	0.59	0.72	0.82	0.39	0.39
	WWSS	0.61	0.59	0.57	0.70	0.34	0.47
	WWWS	0.66	0.63	0.55	0.55	0.41	0.49
	(c) Six equally spaced LLRSE structures						
Weak	WSSSSS	0.88	0.75	0.89	1.13	1.22	1.26
	WWSSSS	0.79	0.78	0.98	1.27	1.56	1.54
	WWWSSS	0.85	0.89	1.09	1.39	1.71	1.64
	WWWWSS	0.87	1.02	1.14	1.43	1.73	1.76
	WWWWWS	0.93	1.04	1.10	1.30	1.58	1.55
Strong	WSSSSS	0.55	0.66	0.78	0.77	0.42	0.37
	WWSSSS	0.57	0.55	0.68	0.80	0.37	0.46
	WWWSSS	0.61	0.60	0.57	0.71	0.36	0.48
	WWWWSS	0.64	0.63	0.53	0.61	0.38	0.45
	WWWWWS	0.67	0.64	0.58	0.47	0.45	0.76
	(d) Eight equally spaced LLRSE structures						
Weak	WSSSSSSS	0.92	0.74	0.86	1.06	1.12	1.23
	WWSSSSSS	0.81	0.77	0.94	1.25	1.44	1.46
	WWWSSSSS	0.81	0.80	1.02	1.30	1.64	1.61
	WWWWSSSS	0.84	0.90	1.10	1.41	1.74	1.67
	WWWWWSSS	0.87	1.02	1.14	1.46	1.77	1.75
	WWWWWWSS	0.88	1.07	1.14	1.42	1.72	1.73
	WWWWWWWS	0.96	1.06	1.11	1.25	1.50	1.49
Strong	WSSSSSSS	0.59	0.69	0.79	0.74	0.42	0.38
	WWSSSSSS	0.55	0.60	0.74	0.83	0.40	0.40
	WWWSSSSS	0.58	0.57	0.65	0.79	0.37	0.46
	WWWWSSSS	0.61	0.60	0.57	0.72	0.37	0.49
	WWWWWSSS	0.64	0.63	0.53	0.65	0.38	0.47
	WWWWWWSS	0.66	0.63	0.56	0.54	0.41	0.40
	WWWWWWWS	0.68	0.65	0.59	0.51	0.47	0.50

*W denotes weak and S denotes strong.

be seen how a reduction in yield strength can bring back the period dependency, despite the fact it had been eliminated from the SDOF systems yielding at R_y by the calibration to target ductilities. This alone is sufficient to explain the large effect of period noticed in the case of systems with weaker-than-estimated yield strengths.

6. For the methodology followed herein, the LLRSE yielding at R_y (i.e., the strong one in the case of $0.8R_y$ and R_y, and the weak one in the other cases) will always have the same inelastic response as the SDOF system yielding at R_y when Ω = 1.0, and therefore the LLRSE ductility amplification ratios will always be 1.0 in that particular case. This phenomenon, unique to structures having two LLRSEs, can be accurately predicted by theory, and is explained in detail elsewhere (Bruneau and Mahin 1987).

7. As seen from Figs. 3–6, weak LLRSE ductility amplification ratios tend to increase with larger Ω, while strong LLRSE ductility amplification ratios tend to very slightly decrease accordingly. This can be explained by the lower resistance to angular motion provided by systems with larger Ω values, as illustrated in a following section. Obviously, this increase in weak LLRSE ductility amplification ratios with Ω would not be observed as consistently when looking at the response under a given earthquake excitation, because of the particular characteristics proper to any single earthquake record, but it is a clear trend that can be observed from the presented results for the mean responses to the five earthquake excitations used in this study. Although there is a few instances in Figs. 3 and 5 where the weak element ductility amplification ratios decreases for step increases in Ω, most of these decreases are of negligible magnitudes, and principally occur for low Ω values and large dissimilarities in element yield strengths. As expected, more "exceptions" are found in the cases of mean-plus-one-standard-deviation (Fig. 6), but the observed trend is in no way jeopardized.

TABLE 3. Weak LLRSE ductility amplification ratios for initially symmetric single-story structures having four LLRSE, with T_x = 0.4 s, and LLRSE strength combinations of 1.0 and 1.5 R_y

System characteristics		Ω					
Geometric ratio D_1/D_2	Stiffness ratio K_1/K_2	0.4	0.8	1.0	1.2	1.6	2.0
(a) Reference two-LLRSE structure							
—	—	0.84	0.90	1.00	1.25	1.80	1.94
(b) Outside weak LLRSE ductility amplification ratio — Four LLRSE structure							
2	1/16	0.88	1.03	1.23	1.65	2.22	2.43
	1/8	0.88	1.00	1.18	1.57	1.98	2.21
	1/4	0.85	0.97	1.15	1.49	1.85	1.95
	1/2	0.84	0.92	1.10	1.41	1.74	1.81
	1	0.84	0.89	1.06	1.35	1.65	1.80
	2	0.84	0.88	1.03	1.30	1.73	1.81
	4	0.84	0.88	1.02	1.28	1.75	1.84
3	1/16	1.00	1.14	1.40	1.83	2.39	2.46
	1/8	0.93	1.04	1.35	1.66	2.10	2.10
	1/4	0.86	0.95	1.20	1.52	1.88	1.81
	1/2	0.84	0.90	1.10	1.41	1.73	1.66
	1	0.85	0.88	1.06	1.34	1.64	1.64
	2	0.84	0.87	1.03	1.30	1.63	1.70
	4	0.84	0.87	1.02	1.28	1.70	1.76
4	1/16	1.08	1.21	1.60	1.86	2.46	2.43
	1/8	0.95	1.02	1.40	1.65	2.06	2.02
	1/4	0.85	0.93	1.16	1.50	1.84	1.80
	1/2	0.84	0.88	1.10	1.40	1.71	1.65
	1	0.85	0.86	1.05	1.33	1.62	1.57
	2	0.84	0.86	1.03	1.29	1.60	1.66
	4	0.84	0.87	1.02	1.27	1.69	1.74
(c) Inside weak LLRSE ductility amplification ratio — Four LLRSE structure							
2	1/16	0.84	0.87	1.03	1.19	1.56	1.65
	1/8	0.84	0.86	1.02	1.15	1.41	1.53
	1/4	0.84	0.85	1.01	1.11	1.32	1.39
	1/2	0.84	0.84	0.98	1.08	1.26	1.33
	1	0.85	0.83	0.95	1.05	1.26	1.34
	2	0.84	0.83	0.95	1.03	1.34	1.36
	4	0.84	0.83	0.95	1.02	1.36	1.40
3	1/16	0.84	0.84	1.03	1.10	1.31	1.33
	1/8	0.85	0.82	1.01	1.05	1.22	1.20
	1/4	0.85	0.82	0.94	1.02	1.16	1.14
	1/2	0.85	0.82	0.89	0.99	1.11	1.09
	1	0.85	0.82	0.88	0.96	1.08	1.10
	2	0.85	0.81	0.90	0.95	1.16	1.15
	4	0.85	0.81	0.92	0.94	1.21	1.20
4	1/16	0.85	0.82	1.01	1.03	1.20	1.20
	1/8	0.85	0.82	0.94	0.99	1.12	1.11
	1/4	0.85	0.82	0.88	0.96	1.07	1.06
	1/2	0.85	0.82	0.87	0.94	1.04	1.02
	1	0.85	0.82	0.86	0.92	1.01	1.00
	2	0.85	0.81	0.87	0.91	1.09	1.07
	4	0.85	0.81	0.90	0.90	1.15	1.12

Based on previous observations, the following design recommendation can be formulated: For structures that can be idealized within the restrictions of this study, assuming the yield strength of the LLRSEs are dissimilar and can be estimated, the ductility demand of the weaker LLRSE is expected to exceed by approximately 50% the ductility demand of a SDOF system of similar yield strength, if Ω is larger than 1.2. The designer expecting to limit the ductility demand on structural members in those cases should reduce its target ductility demand by 30% (1/1.5 = 0.67).

Extension of investigation to more elaborate case studies

The above results and observations were obtained from relatively simple structural systems and element models. In

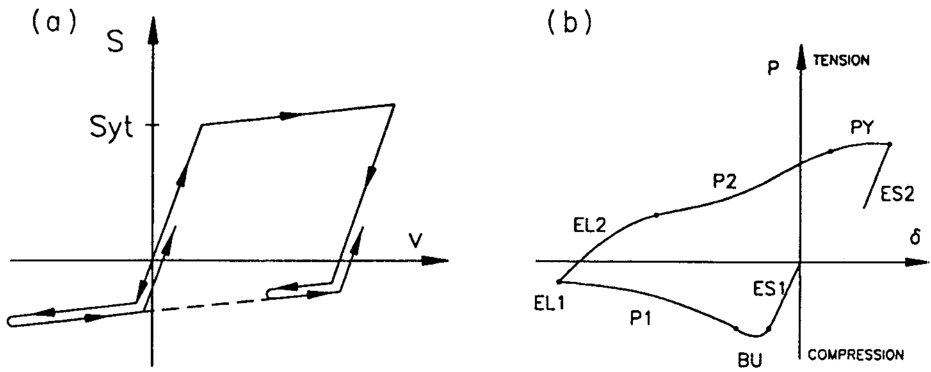

FIG. 7. (*a*) Inelastic brace element model with elastic buckling (yielding in tension, buckling in compression); (*b*) physical brace element model (ES 1, elastic shortening zone; BU, buckling zone; P1, plastic zone; EL1, elastic elongation zone; EL2, elastic elongation zone; P2, plastic zone; PY, yielding zone; ES2, elastic shortening zone).

this section, some more complex structures were investigated to verify whether these previous findings remain valid. Rather than repeating a comprehensive parametric study for many complex systems and all the possible variables, a case-study approach was adopted to investigate the effect of each level of complication individually, using a common reference case as a benchmark when possible.

All initially symmetric single-story cases studied following have an uncoupled translational period of $T_x = 0.4$ s and a LLRSE strength combination of 1.0 and 1.5 R_y. They were subjected to the N–S component of the 1940 El Centro earthquake scaled to produce a target ductility of 4 on the equivalent SDOF systems. Six values of Ω (0.4, 0.8, 1.0, 1.2, 1.6, and 2.0) were used in each case studied.

Single-story multi-element structures

Previously, only structures with two LLRSEs were considered. While these are not uncommon, it was conjectured that results obtained might be overly conservative when applied to structures with more LLRSEs. Consequently, single-story structures with multiple LLRSEs were studied.

As before, the hysteretic bilinear model with 0.5% strain hardening was adopted, Rayleigh damping was set at 2%, and the floor-diaphragm was assumed to be rigid in-plane. Structures with four, six, and eight equally spaced LLRSEs of equal stiffnesses were first considered. In hope of obtaining conservative results while limiting the extent of this survey, all the weak LLRSEs were grouped together, and analysis was conducted for all ensuing possible location of a single transition from weak to strong LLRSEs. The results are reported in Table 2, and compared with those obtained for the simpler two-LLRSE structures. From that comparison, plan redundancy is seen to have little impact on the ductility amplification ratios, or on the previously reported influence of Ω on the response.

Structures with four non-equally spaced LLRSEs were then investigated. For simplicity, these multi-element structures studied were geometrically symmetric about the center of mass, such that each element on one side of the center of mass had an equidistant counterpart of same stiffness on the other side of this center, and LLRSEs sharing identical strength were again grouped on each side of the center of mass. The ratios of the outside and inside LLRSEs distances to the center of mass D_1 and D_2 respectively, and of the corresponding stiffnesses, K_1 and K_2, provide the additional parameters necessary to properly characterize the structures studied. Analysis has been conducted for three values of the D_1/D_2 ratio, seven values of the K_1/K_2 ratio, and all combinations thereof.

The resulting ductility amplification ratios of the outside and inside weak LLRSEs are presented in Table 3. They are found to be almost identical to those obtained for the reference two-LLRSE system. The D_1/D_2 ratio has little influence on the outside weak LLRSE's results, but affect the inside weak LLRSE's values in relation to displacement compatibility. Results also appear insensitive to the K_1/K_2 ratio, except for very low K_1/K_2 values (1/8 and 1/16) where the response is driven by the inside LLRSEs, the outside LLRSEs undergoing large displacements as imposed by the rigid diaphragm without contributing much to the overall structural resistance. Increases of the ductility amplification ratio of the outside LLRSEs up to 60% from the reference two-LLRSEs are noticeable in the $K_1/K_2 = 1/16$ case, which, when compounded to the already high values obtained when Ω is large, could eventually become seriously detrimental to the behavior of those LLRSEs. In that respect, excessively small K_1/K_2 ratios should be avoided, or, alternatively, the exterior elements should be removed from the lateral-load-resistance system and simply verified capable of resisting their share of gravity loads under seismically induced deformations, as indirectly suggested in 4.1.9.1.(9)(d) of the 1990 edition of the National Building Code of Canada (NBCC 1990).

Various element models — single-story structures

Up to this point, all analyses were conducted using a bilinear hysteretic model with 0.5% strain hardening. Other element models will now be considered to determine how the previous findings would thus be affected.

To this end, two different models were chosen to reflect the complex hysteretic behavior associated with braced frames:

1. a simple brace model (Fig. 7*a*) that allows yielding in tension and elastic buckling in compression (Mondkar and Powell 1975); and
2. a more complex physical brace model (Fig. 7*b*) that is capable of a better representation of the true behavior of braces (Ikeda and Mahin 1984).

Only X-braced frames were considered; braces were modelled as not physically connected at their midpoint. As

TABLE 4. LLRSE ductility amplification ratios for initially symmetric two-element braced frame structures having T_x = 0.4 s

System characteristics		Ω					
LLRSE	Strong LLRSE yield (*Y*) and buckling (*B*) strengths	0.4	0.8	1.0	1.2	1.6	2.0
(a) Weak LLRSE having $Y=R_Y$ and $B=R_Y$ — Elastic brace buckling model							
Weak	$Y=1.5R_Y$; $B=1.5R_Y$	0.45	0.46	1.00	0.28	1.50	1.56
	$Y=2.0R_Y$; $B=1.0R_Y$	0.56	0.52	1.00	1.24	1.48	1.65
	$Y=2.5R_Y$; $B=0.5R_Y$	1.10	0.97	1.00	0.89	1.39	1.19
Strong	$Y=1.5R_Y$; $B=1.5R_Y$	0.30	0.30	0.32	0.25	0.22	0.28
	$Y=2.0R_Y$; $B=1.0R_Y$	0.23	0.22	0.22	0.19	0.18	0.23
	$Y=2.5R_Y$; $B=0.5R_Y$	0.41	0.47	0.35	0.39	0.20	0.21
(b) Weak LLRSE having $Y=1.25R_Y$ and $B=0.75R_Y$ — Elastic brace buckling model							
Weak	$Y=1.875R_Y$; $B=1.125R_Y$	0.92	1.03	1.00	1.60	1.35	1.22
	$Y=2.25R_Y$; $B=0.75R_Y$	0.82	0.79	1.00	1.21	1.33	1.20
	$Y=2.625R_Y$; $B=0.375R_Y$	0.79	0.87	1.00	1.25	1.86	1.55
Strong	$Y=1.875R_Y$; $B=1.125R_Y$	0.66	0.67	0.75	0.37	0.24	0.25
	$Y=2.25R_Y$; $B=0.75R_Y$	0.46	0.60	0.57	0.25	0.24	0.25
	$Y=2.625R_Y$; $B=0.375R_Y$	0.52	0.55	0.44	0.41	0.33	0.28
(c) Weak LLRSE having $Y=1.50R_Y$ and $B=0.50R_Y$ — Elastic brace buckling model							
Weak	$Y=2.25R_Y$; $B=0.75R_Y$	0.62	0.66	1.00	1.24	1.27	1.18
	$Y=2.50R_Y$; $B=0.50R_Y$	0.88	0.71	1.00	1.43	1.70	1.58
	$Y=2.75R_Y$; $B=0.25R_Y$	1.01	1.01	1.00	1.67	1.98	1.99
Strong	$Y=2.25R_Y$; $B=0.75R_Y$	0.50	0.55	0.45	0.47	0.40	0.24
	$Y=2.50R_Y$; $B=0.50R_Y$	0.58	0.68	0.40	0.37	0.32	0.38
	$Y=2.75R_Y$; $B=0.25R_Y$	0.62	0.60	0.52	0.43	0.31	0.24
(d) Weak LLRSE having $Y=1.25R_Y$ and $B=0.75R_Y$ — Physical brace buckling model							
Weak	$Y=1.50R_Y$; $B=0.50R_Y$	1.08	1.13	0.99	1.14	1.65	1.35
Strong	$Y=1.50R_Y$; $B=0.50R_Y$	0.82	0.88	0.89	0.87	0.57	0.71

before, the results obtained for the initially symmetric structures are compared with those from the corresponding SDOF systems having an identical braced-frame hysteretic model. All steps of the aforementioned methodology were repeated, including the calibration of earthquake excitation to levels producing a displacement ductility demand of 4 on the equivalent SDOF braced-frame systems.

For the elastic buckling brace element model, various combinations of compression buckling strength and tension yield strength were investigated. For the cases studied, the sum of those strengths was constrained to be identical. Three weak LLRSE yield and buckling strength combinations were investigated: (*i*) yield = $1.0R_y$ and buckling = $1.0R_y$, (*ii*) yield = $1.25R_y$ and buckling = $0.75R_y$, and (*iii*) yield = $1.5R_y$ and buckling = $0.5R_y$, representative of braced-frames built with members of short, medium, and large slenderness ratios respectively. To each of those weak LLRSEs were paired various strong LLRSEs having a sum of yield and buckling strengths 50% larger, creating unbalanced strength in plan.

For the physical brace buckling model, the investigation was limited to cases for which the weak LLRSE yield and buckling strengths were yield = $1.25R_y$ and buckling = $0.75R_y$, and the strong LLRSE strengths were yield = $1.5R_y$ and buckling = $0.5R_y$, corresponding to braced-frames made of diagonal elements having intermediate and large slenderness ratios respectively. As a consequence of the force–displacement model selected, the sum of the yield and buckling strengths of each frame varies as a function of the magnitude of displacements.

The yield displacement needed to calculate displacement ductility was taken as the value at the onset of first yielding of the brace in tension. The ductility amplification ratios resulting from these analyses are presented in Table 4. As revealed by the results in Table 4, conclusions reached using the bilinear hysteretic model can be conservatively extended to other constitutive models, provided the comparison follows the above guidelines. However, an accurate assessment of the modifications in structural behavior attributable to the variability of braces slenderness ratios amongst the various LLRSEs cannot be inferred from this limited study, and is beyond the scope of this work.

Multistory structures

Structures with regular configuration for which the floor plan remained the same for each story (equal mass and mass moment of inertia), and where the reduction in stiffness from story to story remained proportional for each LLRSE, were studied. The ratio of second story over first story stiffnesses was 2/3. These monosymmetric structures had two LLRSEs per story, and the LLRSEs on one side of the center of mass had 150% of the strength of the others. The corresponding two uncoupled translational periods were 0.28 and 0.12 s, and the Ωs are referred to, here, only as general indicators of plan geometry similitudes with the previously studied single-story structures. The roof yield displacement

TABLE 5. LLRSE ductility amplification ratios for initially symmetric two-story structures with K_{top}/K_{bottom} = 2/3, and LLRSE strength combinations of 1.0 and 1.5 R_y

System characteristics		Ω*					
LLRSE	Ductility amplification ratio considered	0.4	0.8	1.0	1.2	1.6	2.0
Weak	Interstory 0–1	0.41	0.81	0.99	1.09	1.14	1.28
	Interstory 1–2	0.91	0.72	1.00	1.05	1.36	1.23
	Total 0–2	0.83	0.86	0.99	1.30	1.20	1.26
Strong	Interstory 0–1	0.37	0.33	0.37	0.33	0.28	0.20
	Interstory 1–2	0.66	0.82	0.63	0.61	0.27	0.29
	Total 0–2	0.75	0.79	0.76	0.71	0.44	0.37

*Ω as a general indicator of geometric similitude only.

was defined as the sum of each story's LLRSE yield displacement.

LLRSEs were modelled as being bilinear hysteretic with 0.5% strain hardening. Rayleigh damping of 2% was chosen at the first and last periods of the multistory system, which implies that the intermediate periods had a somewhat lower damping. No target ductility was fixed for the symmetric structures in these analyses. Instead, the N–S component of the 1940 El Centro earthquake, with peak acceleration scaled to 0.5g, was used.

Each initially symmetric structure with strength eccentricities in plan was compared with a similar two-story stiffness and strength symmetric structure. The total displacement ductilities, as well as each of the interstory displacement ductilities, have been calculated for each structure, and the resulting ductility amplification ratios are presented in Table 5.

Again, as seen from the above results, the observations previously made appear equally applicable to simple multistory structures with regular configuration, as per the above basis of comparison. In fact, the results obtained from the two-LLRSE single-story simple structures apparently could be used to conservatively predict the multistory response.

Consideration of rotational inertia

The sensitivity of ductility amplification ratios on the Ω value has been highlighted above. This section concentrates on illustrating the mechanisms that produce this dependency.

In initially symmetric structures, where the normalized eccentricity (e/r) is zero, it is the Ω factor that reflects the significance of r, the radius of gyration of the floor plan, here taken around the center of mass. This dimensional parameter, a physical representation of the mass distribution around the center of mass, is related to the selected floor plan configuration; although this property in practice is mostly unalterable by the engineer, the effects of variation in radius of gyration on the response of the structures at hand are of interest.

For a given floor translational mass (m), a reduction in r will reduce the rotational inertia (mr^2), and will simultaneously produce an increase in Ω as

$$[13] \quad \Omega = \omega_\theta/\omega_x = (K_\theta m)/(mr^2K_x) = K_\theta/(K_x r^2)$$

Recalling that an initially symmetric structure will respond in a translational manner until yielding of one of the LLRSEs, it becomes obvious that the rotational inertia of the floor plan provides an effective inertia (or resistance) against the introduction of torsional movement during that interval when the instantaneous physical properties of the structure provide a temporary mismatch between the centers of stiffness and centers of mass. If the rotational inertia is very small (large Ω), it is easy to produce a rotational movement as there is little resistance to the induction of angular motion. In the opposite fashion, if the rotational inertia is large (small Ω), considerable inertial resistance to angular motion exists and very little of it may develop.

This phenomenon is illustrated in Fig. 8. In that case, some initially symmetric structures of uncoupled translational period T_x = 0.1 s, having one LLRSE yielding at 80% of the reference yield strength R_y, and the other at R_y, have been analyzed under the N–S component of the 1940 El Centro earthquake record scaled such as to produce a ductility demand of 4 on its equivalent SDOF system (other element model properties as per defined previously). Time-histories of translational (Figs. 8*b*–8*d*) and rotational (Figs. 8*e*–8*g*) responses are presented for systems having Ω values of 0.4, 1.0, and 1.6, along with the time-history response of the equivalent SDOF system (Fig. 8*a*) for comparison purposes. Note that plots of angular motion time histories are at different amplitude scales.

Not only is it obvious from this figure that the structures with lower radii of gyration (thus lower mr^2 and higher Ω) are excited into a larger angular motion, but the transient nature of the inelastic torsional response of initially symmetric systems is well illustrated. Torsional motions are rapidly damped out once the structure returns to a state of balanced yielding or non-yielding, and the torsional frequency of the system (as well as its damping characteristics, of course) greatly influence the rapidity with which those movements are attenuated.

Implications on Canadian design practice

The National Building Code of Canada (NBCC 1990), realizing that many factors and uncertainties in the modelling of the structure and of the excitation may induce torsion even in apparently symmetric structures, requires the consideration of a minimum accidental eccentricity of 0.10D_n, where D_n is the dimension of the building perpendicular to the direction of excitation. As a result of this design criterion, the LLRSEs of symmetric structures are designed to a slightly larger strength than required for a purely sym-

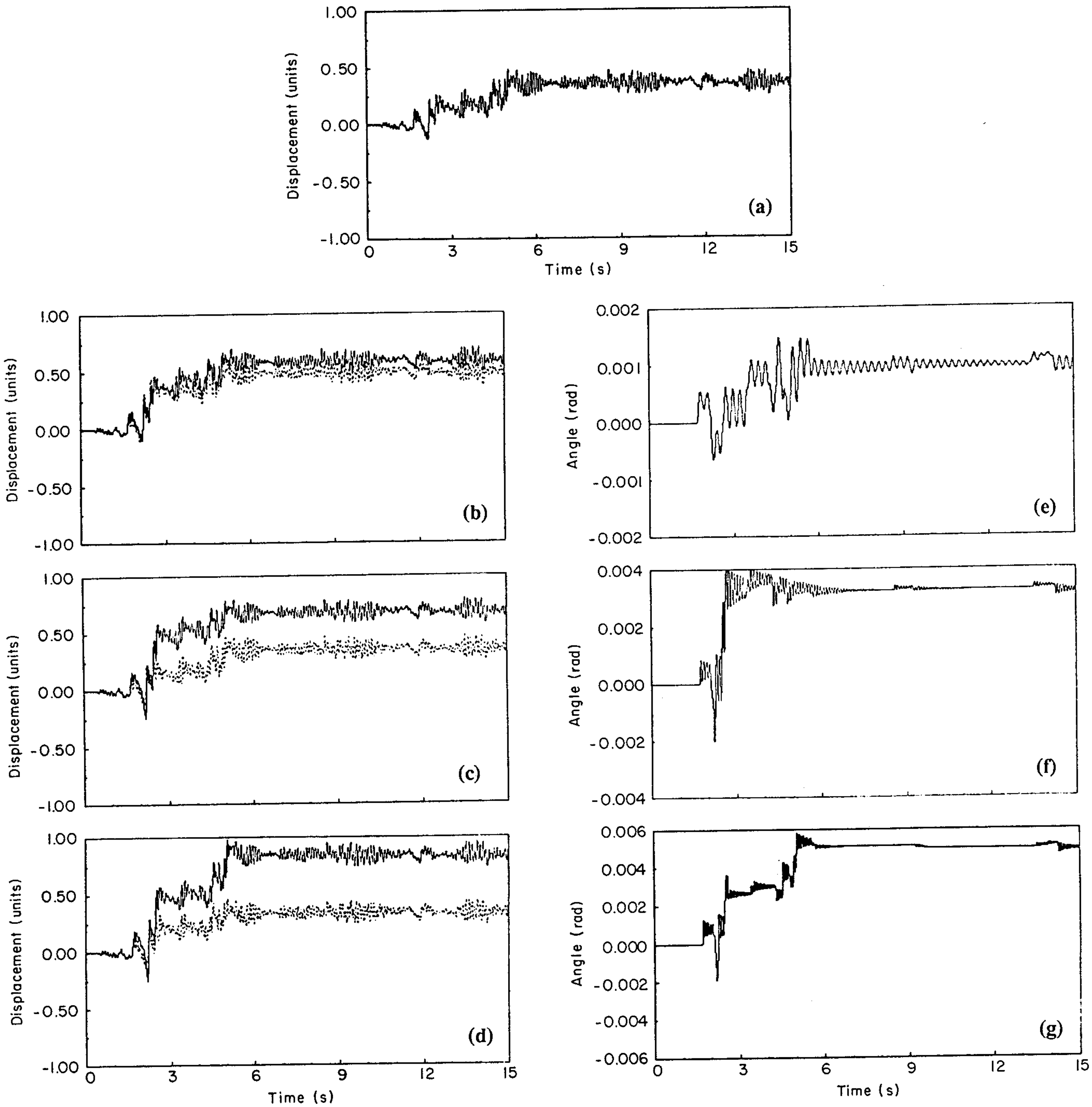

FIG. 8. Inelastic time histories for systems with $T_x = 0.1$ s and $\mu_T = 4$: (*a*) SDOF response; (*b–g*) initially symmetric structures with strength combination of 0.8 and 1.0 R_y, for $\Omega = 0.4$, 1.0, and 1.6 ((*b–d*), translational response; (*e–g*), rotational response).

metric system, to hopefully offset the otherwise larger ductility demand due to these accidental effects. Thus, the intent of this mandatory minimum eccentricity requirement is to ensure that ductility demands of individual LLRSE in the real and imperfect structure do not unduly exceed the levels intended in the idealized design model.

The findings reported herein can also be appraised in light of this design requirement. For a simple two-LLRSE single-story structure, the accidental eccentricity clause of the NBCC would directly result in a 20% increase in the LLRSEs strength. The strength of each of the LLRSEs of this stronger structure, still symmetric in a modelling sense, can be said to be 1.0 and 1.0 R_y for the sake of comparison with the above results. The ductility demand of this structure will vary as a function of its dynamic characteristics and that of the earthquake excitation.

There is an implicit attempt in building codes' seismic design equations to make uniform this demand, in an average sense considering the large variability inherent to individual seismic records, for a given material and type of lateral-load-resisting structural system (LLRSS). The new format of the NBCC 1990 minimum lateral seismic base shear equation,

$$[14] \quad V = V_e U/R$$

similarly aims at providing a uniform level of protection to all structures, through the R/U ratio, where R is a factor indicative of the capability of a structure to dissipate energy through inelastic deformation, U is a calibration factor to past practice, and V_e is equivalent to the seismically induced base shear force should response be purely elastic.

However, the above 20% increase in strength due to compliance with the NBCC accidental eccentricity requirements will results in a nonuniform decrease in ductility demand, even in an average sense. Inelastic constant ductility response spectra (Mahin and Lin 1983) need be consulted to determine the magnitude of new ductility demand, $(R/U)^*$, for this symmetric structure.

Based on the findings from the study reported herein, in the presence of strength eccentricity when some LLRSEs are strengthened or found to be stronger than needed to satisfy accidental eccentricity code requirements, an increased ductility demand of the weaker LLRSE is anticipated, which is of the order of 50% when Ω is larger than 1.2. Therefore, the spirit of the code will be preserved if $1.5(R/U)^* < R/U$; this is best verified on a case-by-case basis, as $(R/U)^*$ is a function of the type and the dynamic characteristics of the LLRSS as well as the characteristics of a given earthquake excitation. Alternatively, it may be deemed an acceptable additional risk of damage considering other larger variabilities and uncertainties involved in the R/U factor itself.

In the presence of strength eccentricity when some LLRSEs are weaker than needed to satisfy accidental eccentricity code requirements, the anticipated increase in the ductility demand of the weak LLRSE can be estimated as $1.5(R/U)^{**}$ when Ω is larger than 1.2, where $(R/U)^{**}$ is the ductility demand of a SDOF system of strength equal to the weaker LLRSE. The relationship $(R/U)^{**} = (\alpha/1.5)(R/U)^*$ is greatly variable, and it has been demonstrated above that α can be large, especially for small periods and large Ω values, where values of α up to 4.0 were observed for the strength combination $0.8R_y$ and $1.0R_y$. The assessment of whether $\alpha(R/U)^* > R/U$ is beyond the scope of this work.

It is noteworthy that Pekau and Guimond (1990), in their approach to the problem, have studied single-story systems having two LLRSEs and strength combinations of R_y and $(1 - \lambda)R_y$, with λ varying from 0 to 0.4.

Unfortunately, the edge displacement at the flexible edge of the building was selected to be the response parameter of interest; this edge was also constrained to be at a constant distance of 1.5 times the radius of gyration as measured from the center of mass. While this is useful if concerned with cladding anchorage design, sizing of seismic joints, and other peripheral considerations, it does not provide information on ductility demand of the LLRSE which has a direct impact on structural performance and damage. The relationship between edge displacement and ductility demand of the LLRSEs is complex in the dynamic domain, unless the LLRSEs are coincident with the edge of the building. Furthermore, Pekau and Guimond have considered three of the five earthquake records adopted herein, omitting the Parkfield and Paicoma Dam records which were found by the authors to affect considerably the results obtained for structures with strength combination of $0.8R_y$ and $1.0R_y$, particularly for translational periods of 0.2 s and smaller. Similarly, Pekau and Guimond did not consider periods below 0.25 s in their study.

Nonetheless, Pekau and Guimond's study confirms that very large amplification of response will occur on the weaker side of structures having $\lambda > 0$ when compared with perfectly symmetric structures ($\lambda = 0$). However, they observed this amplification to be larger for structures having smaller Ω. This is a consequence of modelling; Pekau and Guimond reduce the value of Ω by moving the LLRSEs closer to the center of mass. Thus, for a given ductility demand on the weak LLRSE, a larger weak-side edge displacement is predictable for a smaller Ω should this edge remain at constant distance from the center of mass. The definition of geometrically equivalent nonlinear inelastic torsionally coupled structures is a topic of its own, some aspects of which have been presented elsewhere (Bruneau and Mahin 1991).

In light of the aforementioned differences, a more quantitative comparison of results between the two studies is unfortunately not possible.

Conclusions

For the simple initially symmetric structures studied, which have unbalanced yield strengths in plan, a transient torsional response is created by the desynchronizing in inelastic element response, despite the existence of symmetry in the elastic domain. The resulting element ductility amplification ratios will remain low provided the ratio of uncoupled frequencies, Ω, is not excessively large (preferably 1.2 and lower), and the yield strength of the weaker element in the initially symmetric structure is not less than the yield strength of the reference SDOF system used as the basis for comparison. This conclusion is seen to remain valid for all translational periods and levels of seismic excitation. The reduction in rotational inertia, and consequently lower resistance to angular motion, accounts for the larger ductility amplification ratios at larger Ω values.

Although some structures having more complex force–displacement relationships, increased redundancy, and simple multiple-story configurations have also been studied by the authors, additional research is desirable to further investigate the torsional response of initially symmetric structures having LLRSEs with dissimilar force–displacement relationships, more complex three-dimensional structural configurations, and concurrent bidirectional seismic input. The study of actual structures that have sustained serious damage as a consequence of inelastic torsional coupling, as well as experimental testing of prototype structures, would also be greatly beneficial.

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